Electroproduction of the $N^*(1535)$ resonance at large momentum transfer

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We report on the first lattice calculation of light-cone distribution amplitudes of the $N^*(1535)$ resonance, which are used to calculate the transition form factors at large momentum transfers using light-cone sum rules. In the region $Q^2 > 2 \text{ GeV}^2$, where the light-cone expansion is expected to converge, the results appear to be in good agreement with the experimental data.

Introduction. — Electroproduction of nucleon resonances has long been recognized as an important tool in the exploration of the nucleon structure at different scales. Transition form factors to nucleon excited states allow one to study the relevant degrees of freedom, the wave function and the interaction between the constituents. Quantum chromodynamics (QCD) predicts [1, 2, 3, 4, 5, 6] that at large momentum transfer the form factors become increasingly dominated by the contribution of the valence Fock state with small transverse separation between the partons. There is a growing consensus that perturbative QCD (pQCD) factorization based on hard gluon exchange is not reached at present energies; however, the emergence of quarks and gluons as the adequate degrees of freedom is expected to happen earlier, at $Q^2 \sim$ a few GeV². In this transition region the form factors can be measured to high accuracy, see e.g. [7], and such measurements are in fact part of the experimental proposal for the 12 GeV upgrade at Jefferson Lab [8].

Theoretical progress in the QCD description of transition form factors has been slow. The major problem is that any attempt at a quantitative description of form factors in the transition region must include soft nonperturbative contributions which correspond to the overlap integrals of the soft wave functions, see, e.g., [9, 10]. In particular, models of generalized parton distributions (GPDs) usually are chosen such that the experimental data on form factors are described by the soft contributions alone, cf. [11, 12, 13]. A subtle point for these semi-phenomenological approaches is to avoid double counting of hard rescattering contributions "hidden" in the modeldependent hadron wave functions or GPD parametrizations. An approach that is more directly connected to QCD is based on the light-cone sum rules (LCSRs) [14, 15]. This technique is attractive because in LCSRs "soft" contributions to the form factors are calculated as an expansion in terms of the momentum fraction distributions of partons at small transverse separations, dubbed distribution amplitudes (DAs), which are the same quantities that enter the calculation in pQCD, and there

is no double counting. Thus the LCSRs provide one with the most direct relation of the hadron form factors and DAs that is available at present, with no other nonperturbative parameters. Unfortunately, with the exception of the $\Delta(1232)$ resonance, up to now there exists almost no information on the DAs of nucleon resonances. Thus pQCD predictions [16, 17] cannot be quantified and the LCSRs cannot be used as well.

Moments of the DAs can, however, be calculated on the lattice. In this work we suggest a synthetic approach combining the constraints on DAs from a lattice calculation with LCSRs to calculate the form factors. As the first demonstration of this strategy we consider the electroproduction of $N^*(1535)$, the parity partner of the nucleon. This is a special case because lattice calculations of baryon correlation functions always yield results for baryons of both parities, $J^P = 1/2^+$ and $J^P = 1/2^-$ (see, e.g., [18, 19]), so in fact the results for $N^*(1535)$ appear to be a byproduct of our calculation of the nucleon DAs [20, 21], to which we refer for further technical details. We find that the shapes of the nucleon and N^* DAs are rather different. A preliminary account of this study was presented in [22]. In this paper we further use our results on the DAs to calculate the helicity amplitudes $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$ for the electroproduction of $N^*(1535)$ in the LCSR approach. In the region $Q^2 > 2 \text{ GeV}^2$, where the light-cone expansion may be expected to converge, the results appear to be in good agreement with the experimental data.

Distribution Amplitudes. — The leading-twist(=3) nucleon (proton) DA can be defined from a matrix element of a nonlocal light-ray operator that involves quark fields of given helicity $q^{\uparrow(\downarrow)} = (1/2)(1 \pm \gamma_5)q$ [23]:

$$\begin{split} \langle 0 | \epsilon^{ijk} \Big(u_i^\uparrow(a_1 n) C \not n u_j^\downarrow(a_2 n) \Big) \not n d_k^\uparrow(a_3 n) | N(P) \rangle \\ &= -\frac{1}{2} f_N \, P \cdot n \not n u_N^\uparrow(P) \! \int \! [dx] \, e^{-iP \cdot n \sum x_i a_i} \, \varphi_N(x_i). \end{split}$$

Here P_{μ} , $P^2=m_N^2$, is the proton momentum, $u_N(P)$ the usual Dirac spinor in relativistic normalization, n_{μ} an

arbitrary light-like vector $n^2=0$ and C the charge-conjugation matrix. The variables x_1,x_2,x_3 have the meaning of the momentum fractions carried by the three valence quarks and the integration measure is defined as $\int [dx] = \int_0^1 dx_1 dx_2 dx_3 \delta(\sum x_i - 1)$. The Wilson lines that ensure gauge invariance are inserted between the quarks; they are not shown for brevity.

The nonlocal operator on the l.h.s. of (1) does not have a definite parity. Thus the same operator couples also to $N^*(1535)$ and one can define the corresponding leading-twist DA as

$$\begin{split} \langle 0 | \epsilon^{ijk} \Big(u_i^\uparrow(a_1 n) C \not n u_j^\downarrow(a_2 n) \Big) \not n d_k^\uparrow(a_3 n) | N^*(P) \rangle \\ &= \frac{1}{2} f_{N^*} \, P \cdot n \not n u_{N^*}^\uparrow(P) \! \int \! [dx] \, e^{-iP \cdot n \sum x_i a_i} \, \varphi_{N^*}(x_i) \,, \end{split}$$

where, of course, $P^2=m_{N^*}^2$. The normalization constants f_N and f_{N^*} are defined as

$$\langle 0|\epsilon^{ijk}(u_iC\psi u_j)(0)\gamma_5\psi d_k(0)|N(P)\rangle = f_N P \cdot n \psi u_N(P)$$

$$\langle 0|\epsilon^{ijk}(u_iC\psi u_j)(0)\gamma_5\psi d_k(0)|N^*(P)\rangle$$

$$= f_{N^*}P \cdot n\gamma_5\psi u_{N^*}(P). \tag{2}$$

On the lattice one can calculate moments of the DA

$$\varphi^{lmn} = \int [dx] x_1^l x_2^m x_3^n \varphi(x_i),$$

which are related to matrix elements of local three-quark operators with covariant derivatives, see [21] for details. The normalization is such that $\varphi^{000}=1$.

There exist three independent subleading twist-4 distribution amplitudes $\Phi_4^{N^*}$, $\Psi_4^{N^*}$, $\Xi_4^{N^*}$ (as for the nucleon). They can be defined as (cf. [23, 24])

$$\begin{split} &\langle 0|\epsilon^{ijk} \Big(u_i^\uparrow(a_1n)C \not\!\!/ u_j^\downarrow(a_2n)\Big) \not\!\!/ d_k^\uparrow(a_3n)|N^*(P)\rangle \\ &= \frac{1}{4}\,P\cdot n \not\!\!/ u_{N^*}^\uparrow(P)\!\!\int\! [dx]\,e^{-iP\cdot n\sum x_i a_i} \\ &\quad \times \Big[f_{N^*}\Phi_4^{N^*,WW}(x_i) + \lambda_1^*\Phi_4^{N^*}(x_i)\Big]\,, \\ &\langle 0|\epsilon^{ijk} \Big(u_i^\uparrow(a_1n)C \not\!\!/ \gamma_\perp \not\!\!/ u_j^\downarrow(a_2n)\Big)\,\gamma^\perp \not\!\!/ d_k^\uparrow(a_3n)|N^*(P)\rangle \\ &= -\frac{1}{2}\,P\cdot n \not\!\!/ m_{N^*}u_{N^*}^\uparrow(P)\!\!\int\! [dx]\,e^{-iP\cdot n\sum x_i a_i} \\ &\quad \times \Big[f_{N^*}\Psi_4^{N^*,WW}(x_i) - \lambda_1^*\Psi_4^{N^*}(x_i)\Big]\,, \\ &\langle 0|\epsilon^{ijk} \Big(u_i^\uparrow(a_1n)C \not\!\!/ m u_j^\uparrow(a_2n)\Big) \not\!\!/ d_k^\uparrow(a_3n)|N^*(P)\rangle \\ &= \frac{\lambda_2^*}{12}\,P\cdot n \not\!/ m_{N^*}u_{N^*}^\uparrow(P)\!\!\int\! [dx]\,e^{-iP\cdot n\sum x_i a_i}\,\Xi_4^{N^*}(x_i)\,, \end{split}$$

where $\Phi_4^{N^*,WW}(x_i)$ and $\Psi_4^{N^*,WW}(x_i)$ are the so-called Wandzura-Wilczek contributions, which can be expressed in terms of the leading-twist DA [24]. The two new normaliza-

		Asympt.	N	$N^{\star}(1535)$
•	f_N		3.234(63)(86)	4.544(117)(223)
	$-\lambda_1$		35.57(65)(136)	37.55(101)(768)
	λ_2		70.02(128)(268)	191.9(44)(391)
	φ^{100}	$\frac{1}{3} \simeq 0.333$	0.3999(37)(139)	0.4765(33)(155)
	φ^{010}	$\frac{1}{3} \simeq 0.333$	0.2986(11)(52)	0.2523(20)(32)
	φ^{001}	$\frac{1}{3} \simeq 0.333$	0.3015(32)(106)	0.2712(41)(136)
	φ^{200}	$\frac{1}{7} \simeq 0.143$	0.1816(64)(212)	0.2274(89)(307)
	φ^{020}	$\frac{1}{7} \simeq 0.143$	0.1281(32)(106)	0.0915(45)(224)
	φ^{002}	$\frac{1}{7} \simeq 0.143$	0.1311(113)(382)	0.1034(160)(584)
	φ^{011}	$\frac{2}{21} \simeq 0.095$	0.0613(89)(319)	0.0398(132)(497)
	φ^{101}	$\frac{2}{21} \simeq 0.095$	0.1091(41)(152)	0.1281(56)(131)
	φ^{110}	$\frac{2}{21} \simeq 0.095$	0.1092(67)(219)	0.1210(101)(304)

TABLE I: The normalization constants (in units of $10^{-3}~{\rm GeV^2}$) and moments of the DAs obtained from QCDSF/DIK configurations at $\beta=5.40$ for the nucleon (N) and $N^\star(1535)$ at $\mu_{\overline{MS}}^2=1~{\rm GeV^2}$. The first error is statistical, the second error represents the uncertainty due to the chiral extrapolation and renormalization. The systematic error should be considered with caution.

tion constants are given by the local matrix elements

$$\langle 0|\epsilon^{ijk}(u_{i}C\gamma_{\mu}u_{j})(0)\gamma_{5}\gamma^{\mu}d_{k}(0)|N^{*}(P)\rangle$$

$$=\lambda_{1}^{*}m_{N^{*}}\gamma_{5}u_{N^{*}}(P),$$

$$\langle 0|\epsilon^{ijk}(u_{i}C\sigma_{\mu\nu}u_{j})(0)\gamma_{5}\sigma^{\mu\nu}d_{k}(0)|N^{*}(P)\rangle$$

$$=\lambda_{2}^{*}m_{N^{*}}\gamma_{5}u_{N^{*}}(P).$$

The asymptotic distribution amplitudes (at very large scales) for the nucleon and N^* are the same:

$$\begin{split} \phi^{\mathrm{as}}(x_i) &= 120x_1x_2x_3\,, \quad \Phi_4^{\mathrm{as}}(x_i) = 24x_1x_2\,, \\ \Phi_4^{WW,\mathrm{as}}(x_i) &= 24x_1x_2(1 + \frac{2}{3}(1 - 5x_3))\,, \\ \Psi_4^{WW,\mathrm{as}}(x_i) &= 24x_1x_3(1 + \frac{2}{3}(1 - 5x_2))\,, \\ \Xi_4(x_i) &= 24x_2x_3\,, \quad \Psi_4^{\mathrm{as}}(x_i) = 24x_1x_3\,. \end{split}$$

Baryon states of different parity can be identified in a lattice calculation as those propagating forward or backward in (imaginary) time as long as their momentum vanishes [18, 19]. While vanishing momentum is sufficient for the evaluation of the normalization constants, the higher moments of the DAs require nonzero momentum. In this case the signal in the negative parity channel is contaminated by a contribution of the $J^P=1/2^+$ (nucleon) ground state of the order $\vec{p}^2/(m_N m_{N^*})$ enhanced by the factor $e^{(E_{N^*}-E_N)(T-t)}$, where T is the time extent of our lattice. However, this effect seems to be quite small in our results: Replacing the parity projector $(1/2)(1+\gamma_4)$ by $(1/2)(1+(m_N/E_N)\gamma_4)$ [18] changes the first (second) moments of the N^* DAs by 1% (5%), which is well below the statistical error. In principle, there is still a contamination by the $J^P = 1/2^+ N^*(1440)$ (Roper) resonance, but for small momenta this effect is expected to be negligible [18]. Another issue is that in the

physical spectrum there are two $J^P=1/2^-$ resonances, $N^*(1535)$ and $N^*(1650)$, which are close to each other, so that they cannot be distinguished by means of their mass difference in our calculation. Because of the peculiar decay pattern of $N^*(1650)$ we expect, however, that this state has a much smaller coupling to the usual interpolating operators [19]. So our results can be identified with the contribution of $N^*(1535)$ alone. All these questions certainly deserve a further study.

The results of our calculation of the normalization constants f_{N^*} , λ_1^* , λ_2^* and of the first few moments of the leading-twist DA of the $N^*(1535)$ resonance are compared to the similar calculation for the nucleon [21] in Table I. It attracts attention that f_{N^*} is about 50% larger than f_N . This means that the wave function of the three quarks at the origin is larger in the $J^P=1^-$ state than in the $J^P=1^+$ state, which may be counterintuitive. The momentum fraction carried by the u-quark with the same helicity as the baryon itself, φ^{100} , appears to be considerably larger for N^* , indicating that its DA is more asymmetric. These features suggest that the large asymmetry of the nucleon DA observed in QCD sum rule calculations [25, 26, 27] may be due to a contamination of the sum rules by states of opposite parity, which are difficult to separate in this approach.

The calculated moments can be used to model the N^* leading-twist DA as an expansion in orthogonal polynomials corresponding to the contributions of multiplicatively renormalizable operators (in leading order), see [21]. The comparison of such models for N and N^* , obtained using the polynomial expansion to second order and the central values of the lattice parameters, is shown in Fig. 1.

Helicity Amplitudes from LCSRs. — The matrix element of the electromagnetic current $j_{\nu}^{\rm em}$ between spin-1/2 states of opposite parity can be parametrized in terms of two independent form factors, which can be chosen as

$$\langle N^{*}(P')|j_{\nu}^{\text{em}}|N(P)\rangle = \bar{u}_{N^{*}}(P')\gamma_{5}\Gamma_{\nu}u_{N}(P),$$

$$\Gamma_{\nu} = \frac{G_{1}(q^{2})}{m_{N}^{2}}(\not qq_{\nu} - q^{2}\gamma_{\nu}) - i\frac{G_{2}(q^{2})}{m_{N}}\sigma_{\nu\rho}q^{\rho}, \quad (3)$$

where q=P'-P is the momentum transfer. The helicity amplitudes $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$ for the electroproduction of $N^*(1535)$ can be expressed in terms of these form factors [28]:

$$A_{1/2} = e B \left[Q^2 G_1(Q^2) + m_N (m_{N^*} - m_N) G_2(Q^2) \right],$$

$$S_{1/2} = \frac{e}{\sqrt{2}} B C \left[(m_N - m_{N^*}) G_1(Q^2) + m_N G_2(Q^2) \right].$$

Here e is the elementary charge and $B,\,C$ are kinematic factors defined as

$$B = \sqrt{\frac{Q^2 + (m_{N*} + m_N)^2}{2m_N^5(m_{N^*}^2 - m_N^2)}},$$

$$C = \sqrt{1 + \frac{(Q^2 - m_{N^*}^2 + m_N^2)^2}{4Q^2m_{N^*}^2}}.$$

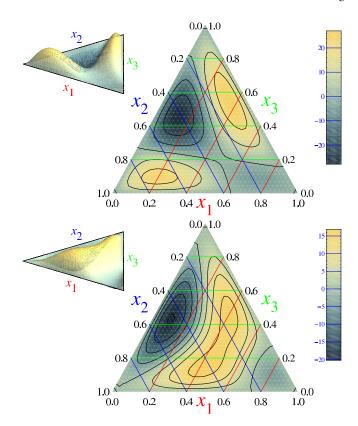


FIG. 1: Barycentric plot of the distribution amplitudes for nucleon (up) and $N^*(1535)$ (down) at $\mu_{\overline{MS}} = 1 \text{GeV}$ using the central values of the lattice results. The lines of constant x_1, x_2 and x_3 are parallel to the sides of the triangle labelled by x_2, x_3 and x_1 , respectively.

The LCSRs are derived from the correlation function

$$\int dx \, e^{-iqx} \langle N^*(P)|T\{\eta(0)j_{\mu}^{\rm em}(x)\}|0\rangle \,,$$

where η is a suitable operator with nucleon quantum numbers, e.g. the Ioffe current [34]. Making use of the duality of QCD quark-gluon and hadronic degrees of freedom through dispersion relations one can write a representation for the form factors appearing in (3) in terms of the DAs of N^* . In leading order, the sum rules for $Q^2G_1(Q^2)/(m_Nm_{N^*})$ and $-2G_2(Q^2)$ have the same functional form as the similar sum rules [14, 15] for the Dirac and Pauli electromagnetic form factors of the proton, with the replacement $m_N \to m_{N^*}$ in the light cone expansion part, and different DAs.

In the present calculation we used a model for the leading-twist DA including first order corrections in the polynomial expansion, asymptotic expressions for the "genuine" twist-4 DAs and the corresponding Wandzura-Wilczek corrections up to twist-6 as given in Ref. [23]. The results are shown in Fig. 2. The shaded areas correspond to the uncertainty in the lattice values as given in Table I. In the region $Q^2 > 2~{\rm GeV}^2$, where the light-cone expansion may be expected to converge, the results appear to be in good agreement with the experimental data.

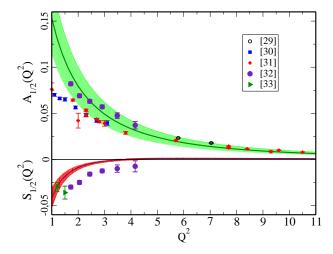


FIG. 2: The LCSR calculation for the helicity amplitudes $A_{1/2}(Q^2)$ and $S_{1/2}(Q^2)$ for the electroproduction of the $N^*(1535)$ resonance using the lattice results from Table I for the lowest moments of the $N^*(1535)$ DAs. The curves are obtained using the central values of the lattice parameters, and the shaded areas show the corresponding uncertainty.

Discussion and Conclusions. — In this work we suggest to calculate transition form factors for nucleon resonances at intermediate momentum tranfer, combining the constraints on DAs from a lattice calculation with LCSRs This approach seems to be especially promising for $N^*(1535)$, the parity partner of the nucleon, because of the relative ease to separate the states of opposite parity on the lattice. The accuracy is expected to increase significantly when calculations with smaller pion masses and on larger lattices become available. This would remove a major source of uncertainties which is due to the chiral extrapolation.

In order to match the accuracy of the lattice results, the LCSR calculations of baryon form factors will have to be advanced to include NLO radiative corrections, as it has become standard for meson decays. For the first effort in this direction see [35]. In addition, one needs a technique for the resummation of "kinematic" corrections to the sum rules that are due to the large masses of the resonances.

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